Universidade do Minho

A Net-based Formal Framework for Causal Loop Diagrams

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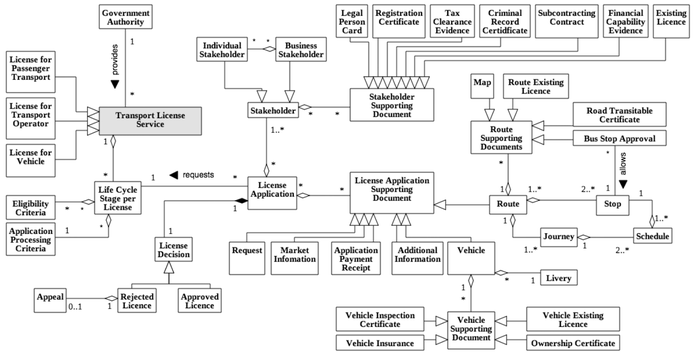
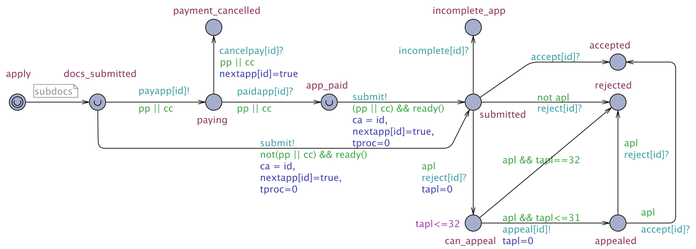
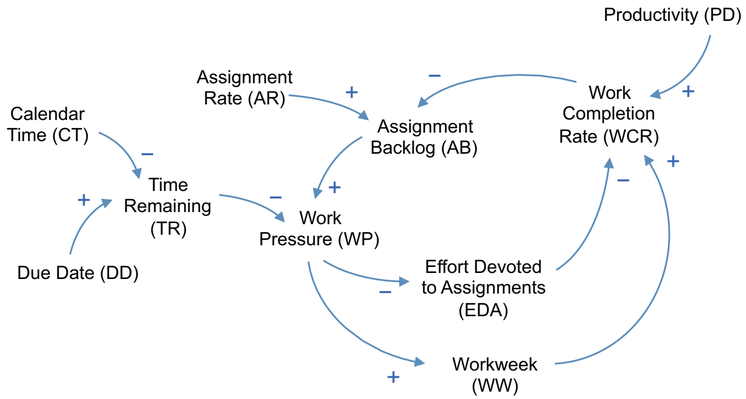
Managing complex systems



Structure

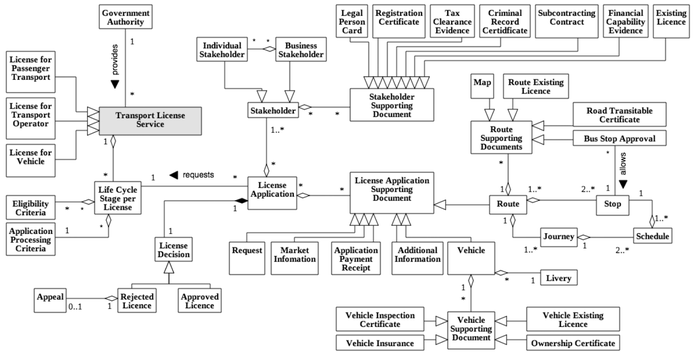


Behaviour



Dynamics

Managing complex systems



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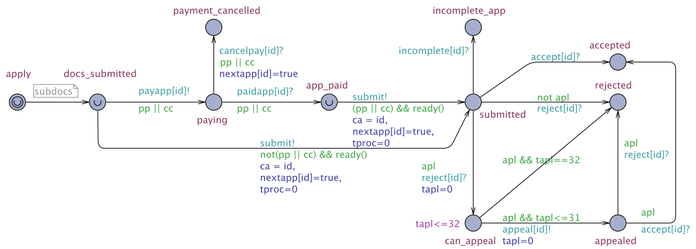

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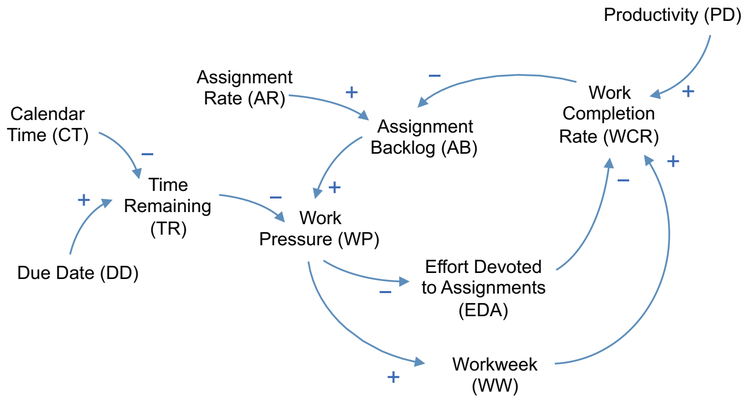
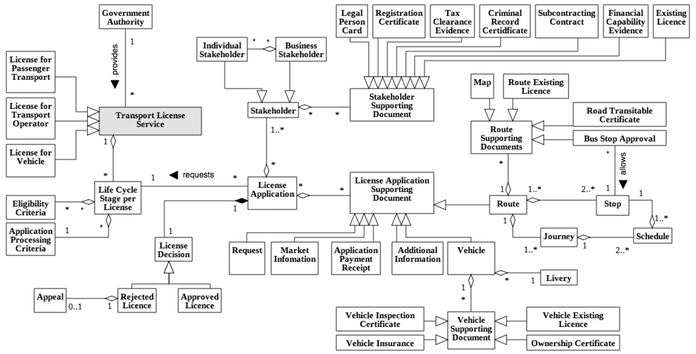


Dynamics

Understand causal links between variables of the system

Managing complex systems



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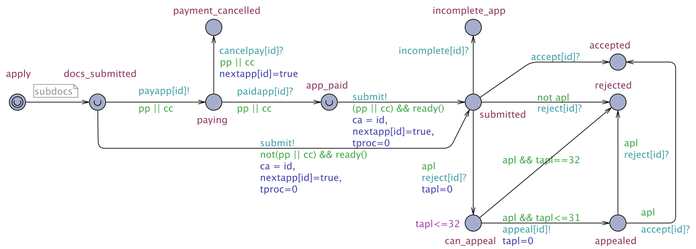
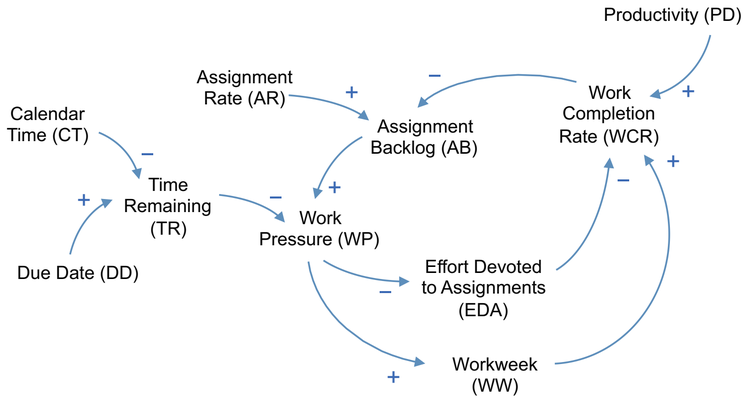


Behaviour

Causal Loop Diagrams (CLD)
(Qualitative approach)



Dynamics

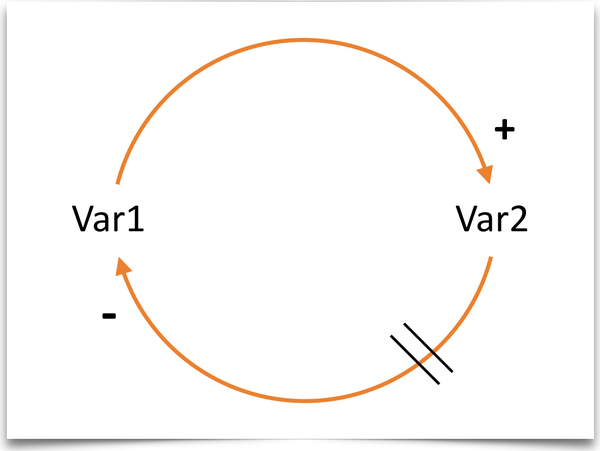
Understand causal links between variables of the system

Causal Loop Diagrams

Illustrates causal links between concepts ⇢ Causes and Effects

Abstracts from quantities

Variables only **Increase** or **Decrease**



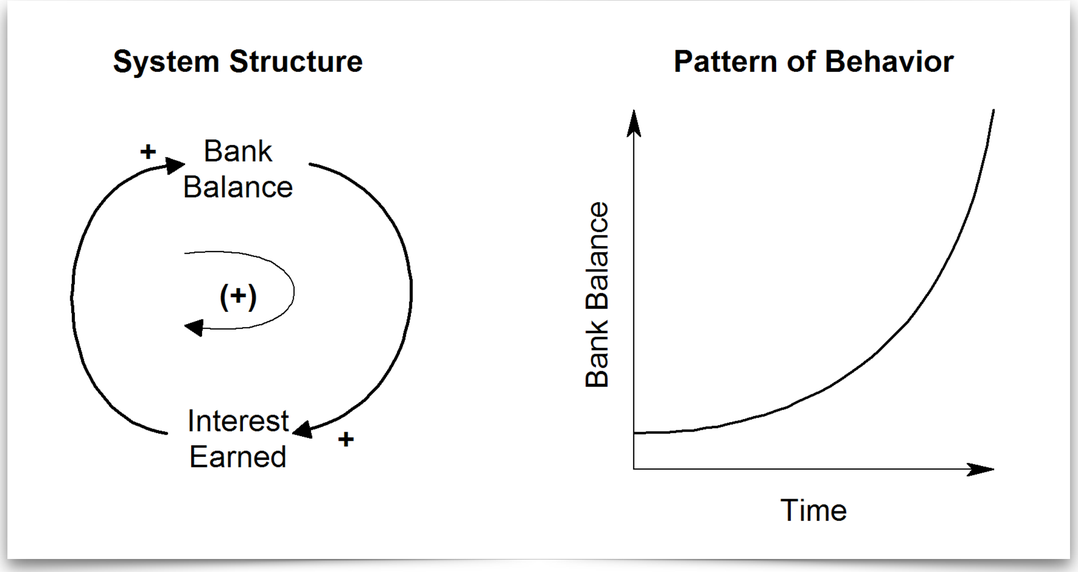
Describes system structure

Links' **polarities**: How the independent variable affects the dependent one?

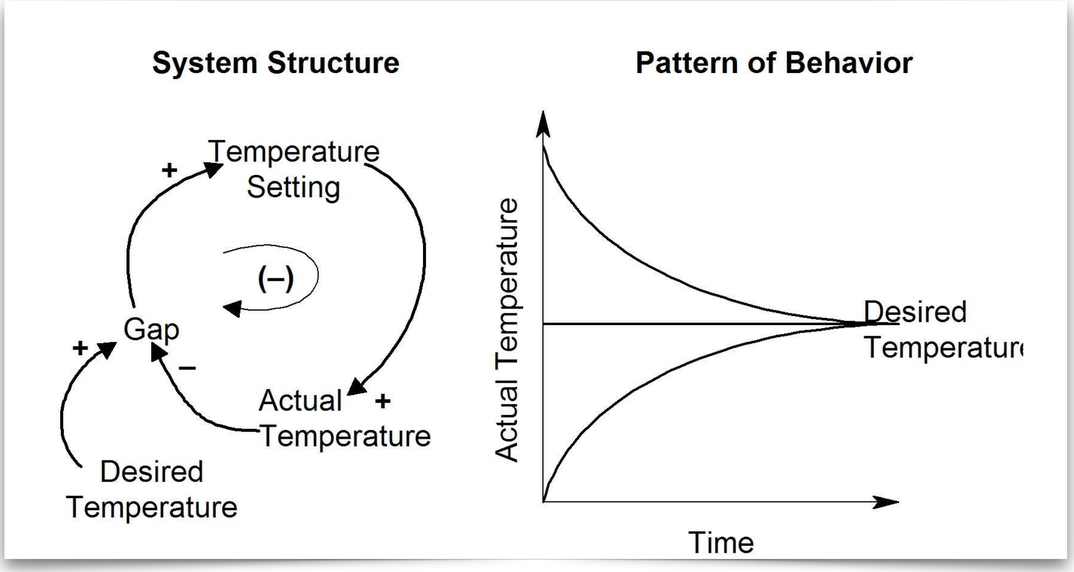
Brings out dynamic behaviour

Causal Loop Diagrams

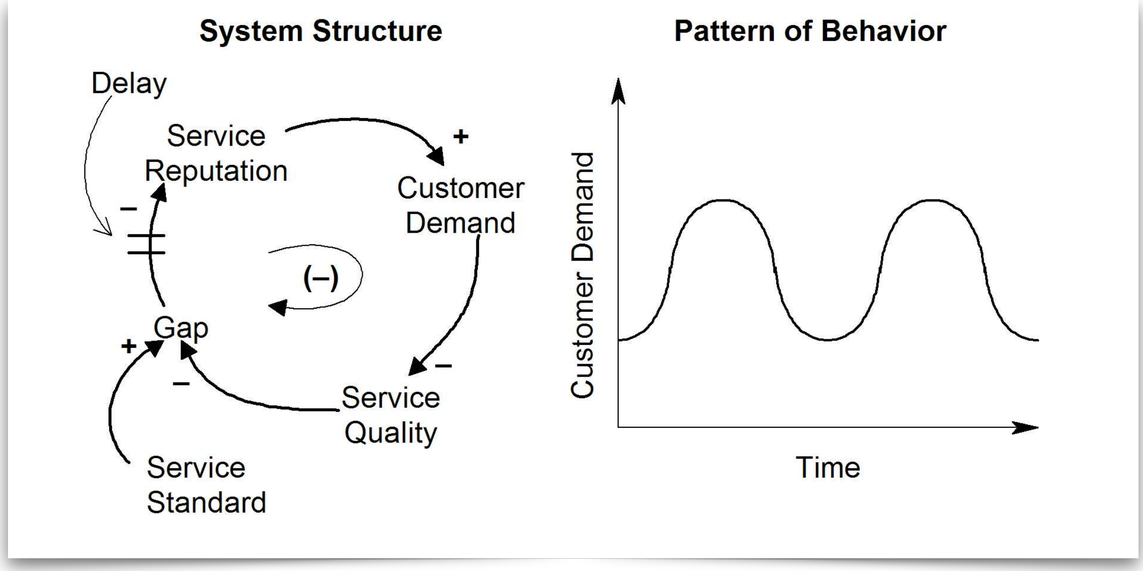
Reinforcing



Balancing



Balancing with delay

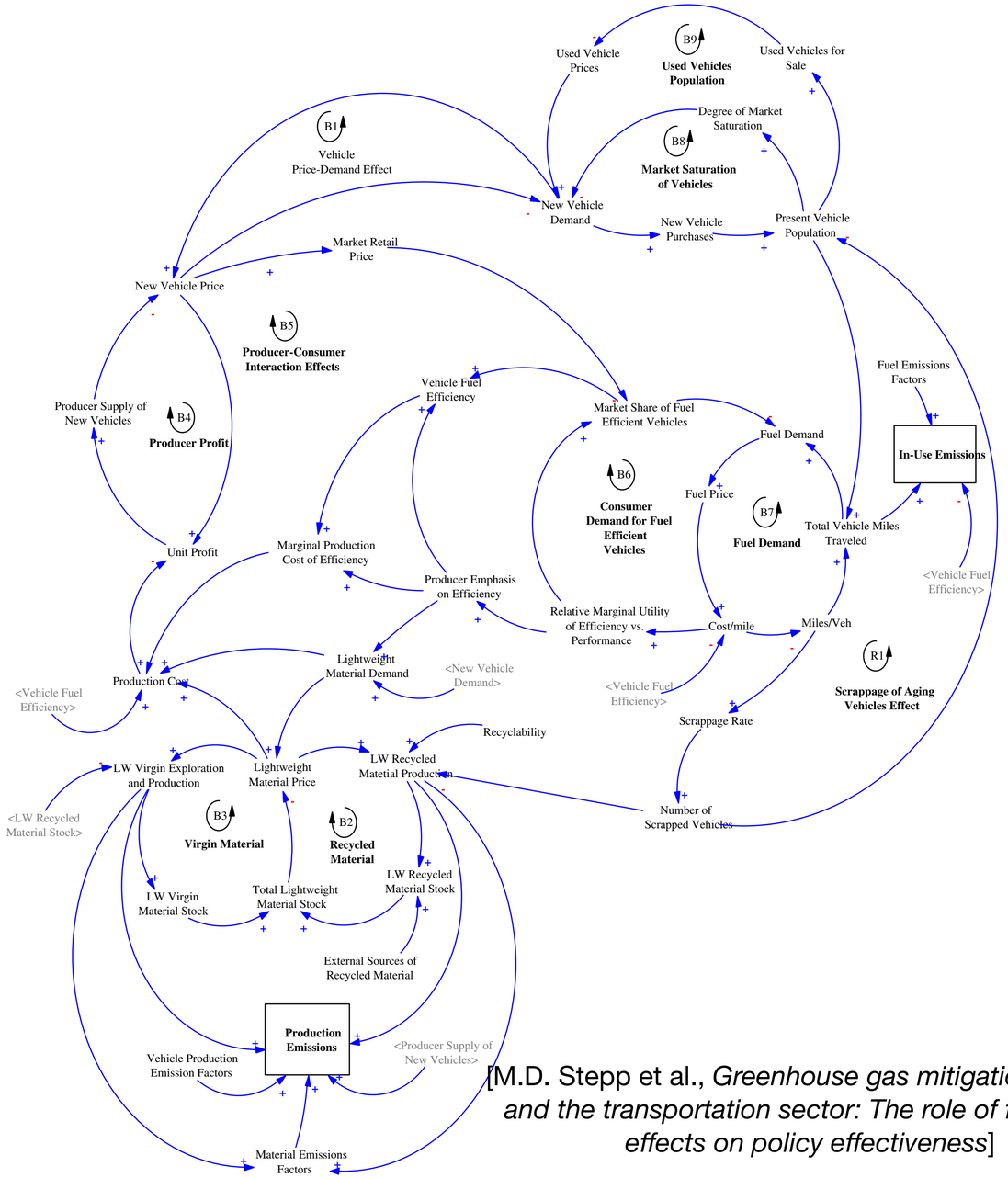


[C.W. Kirkwood, *System Dynamic Methods*]

Causal Loop Diagrams

Challenges

- Complex interactions
- Informal semantics



[M.D. Stepp et al., *Greenhouse gas mitigation policies and the transportation sector: The role of feedback effects on policy effectiveness*]

Causal Loop Diagrams

Challenges

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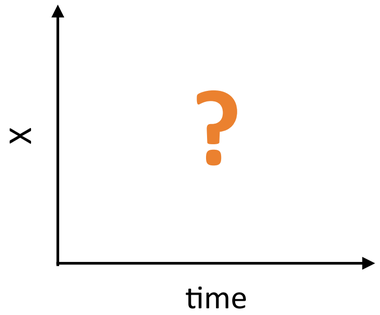
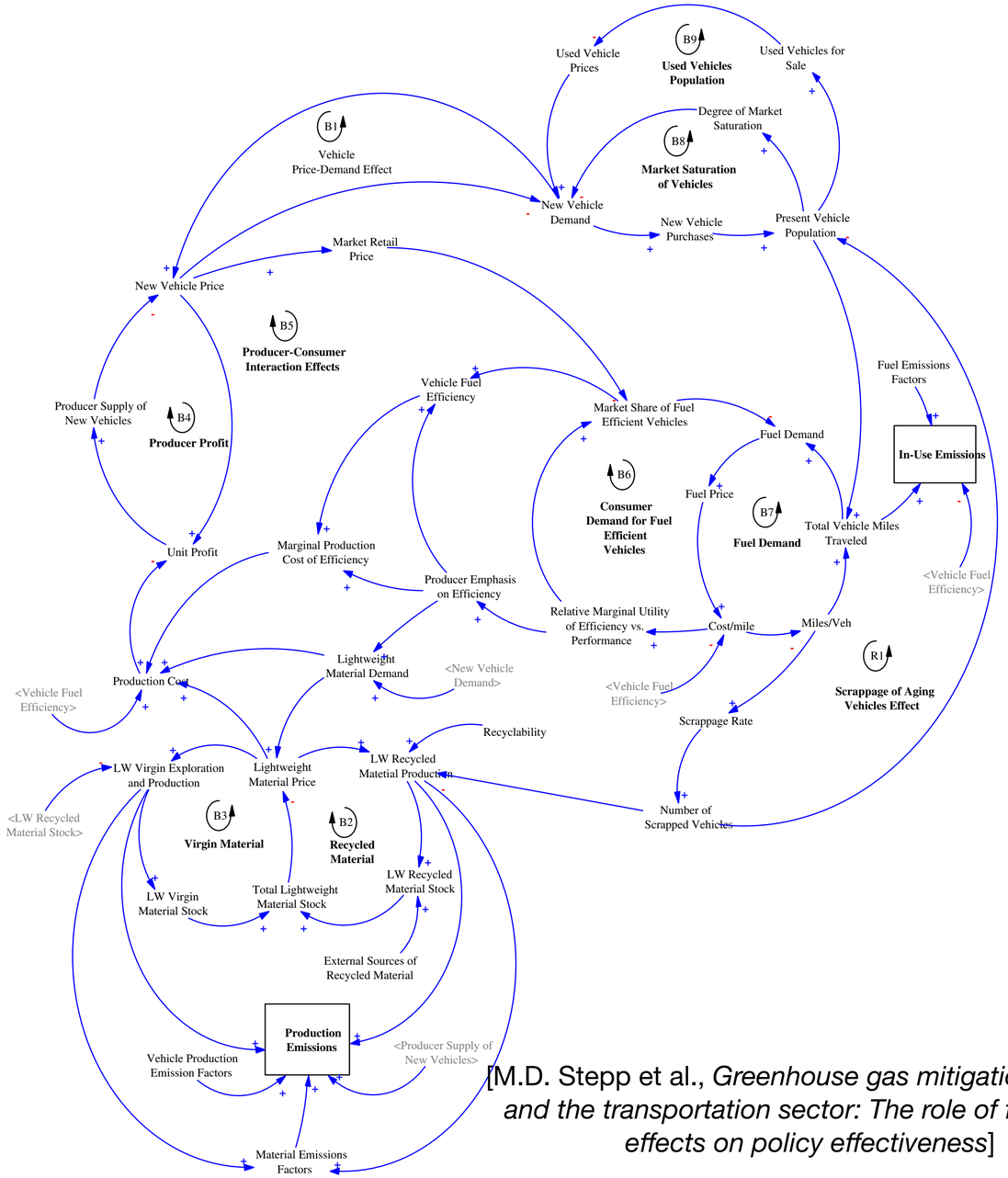
Pattern of behaviour?





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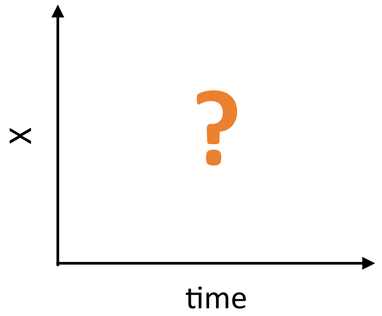
Causal Loop Diagrams

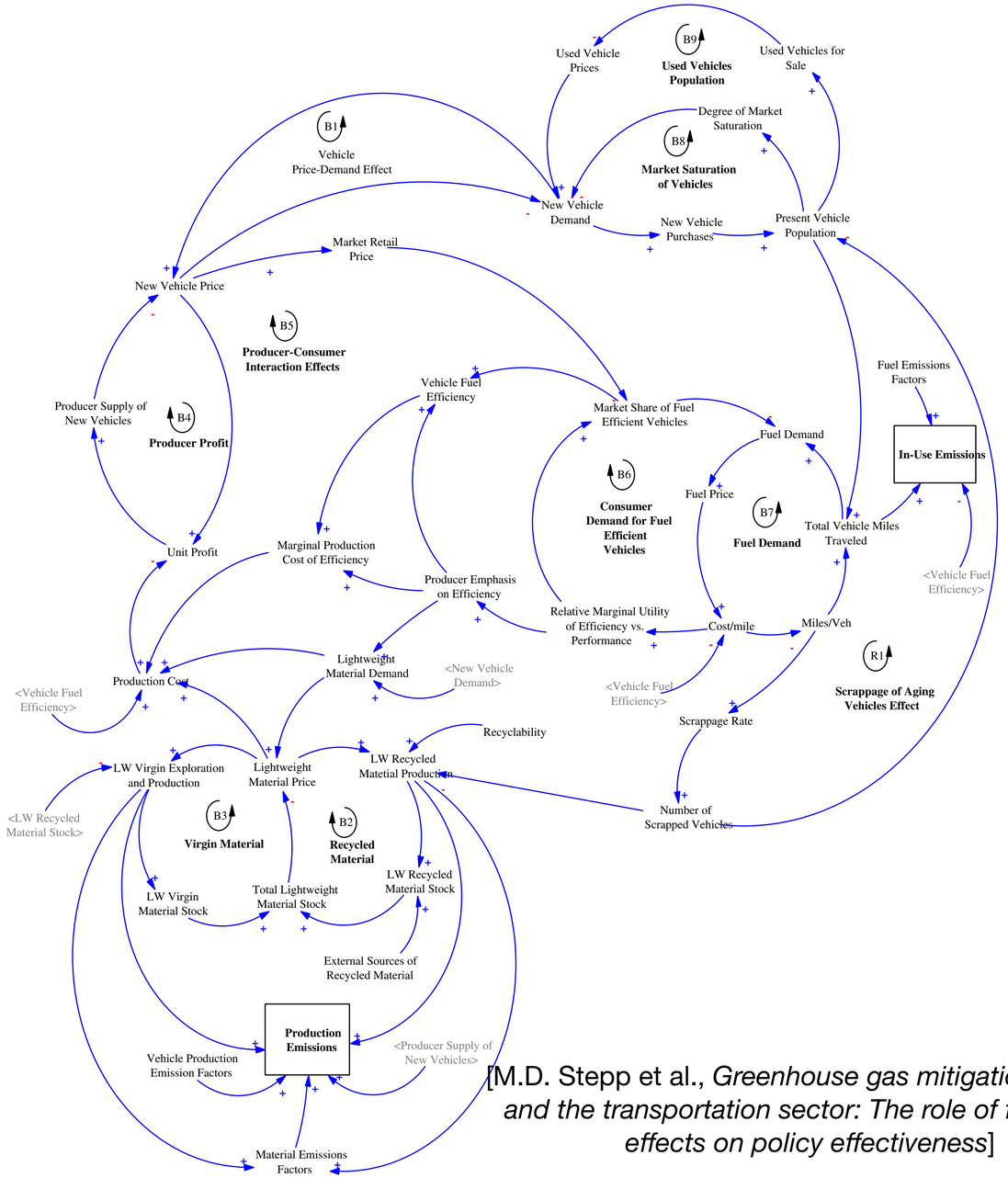
Challenges

- Complex interactions
- Informal semantics

Simulation
(not exhaustive)

Pattern of behaviour?





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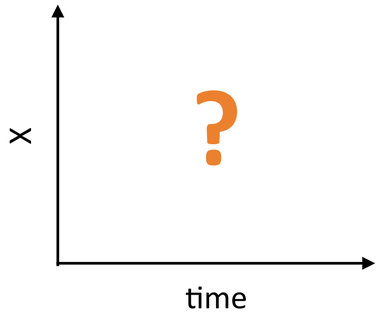
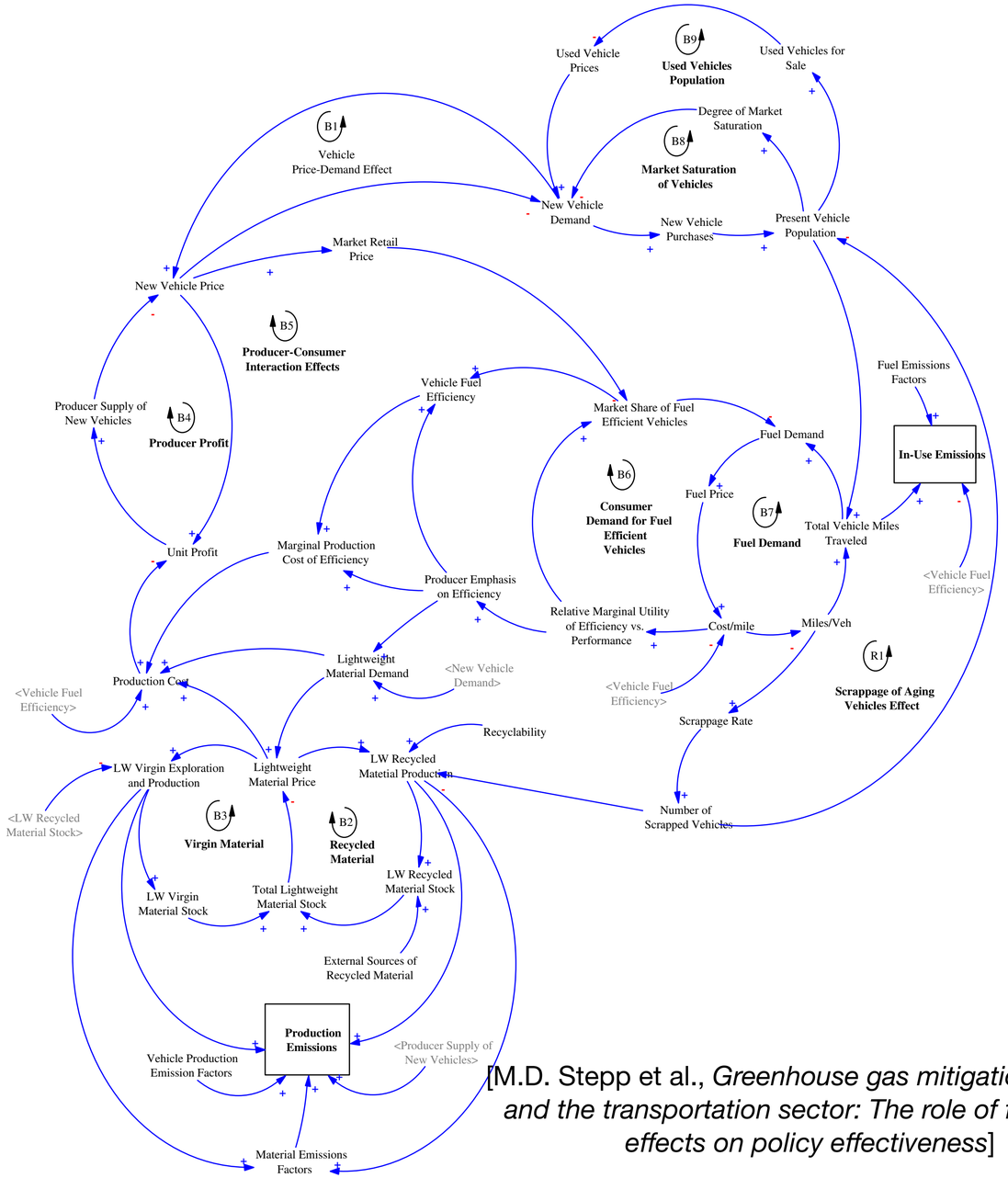


Formal semantics

Formal Analysis
(exhaustive)



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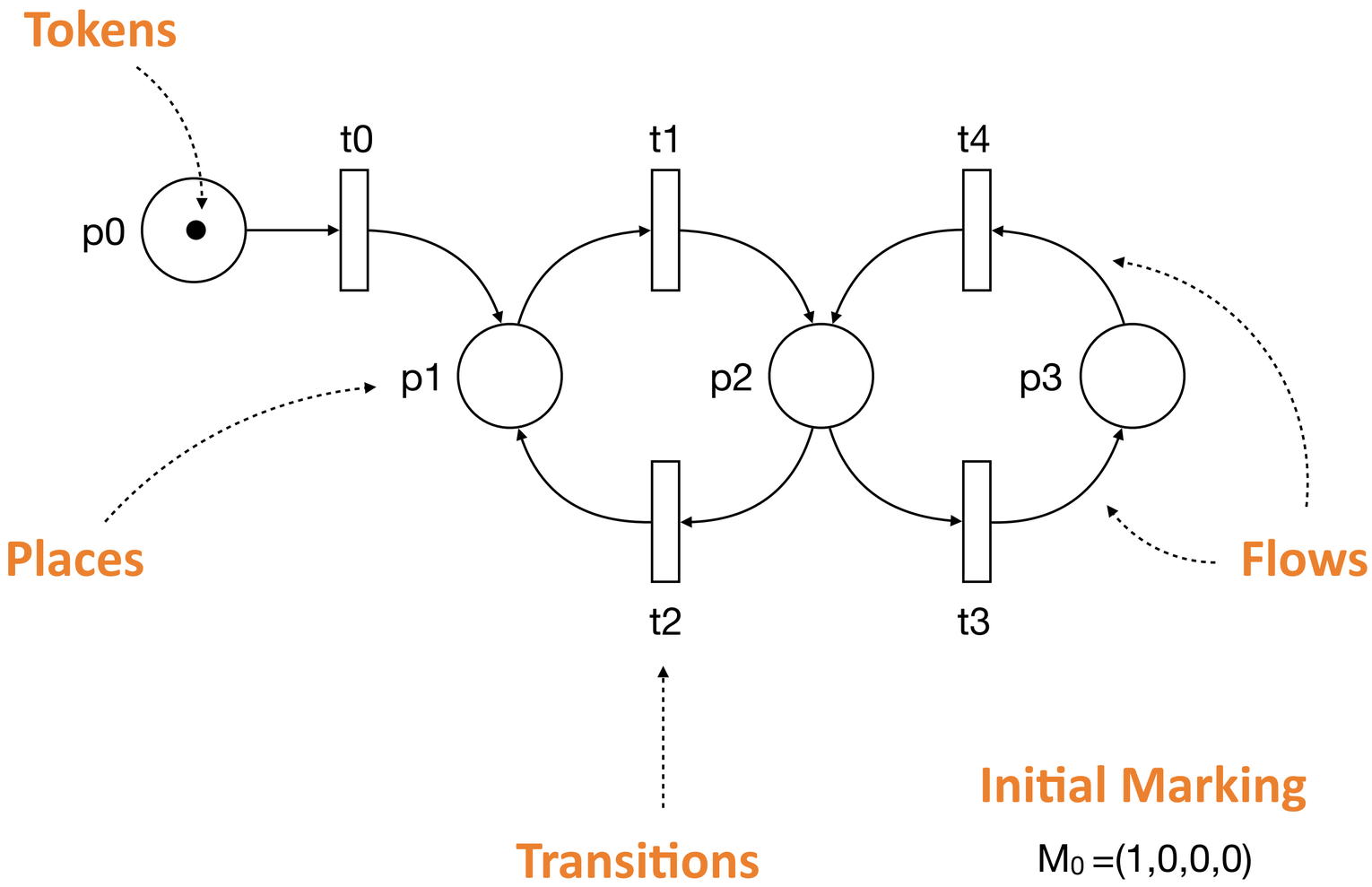
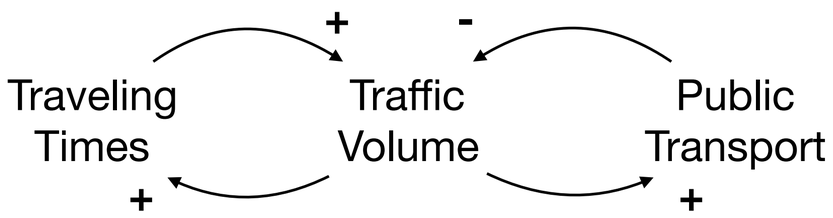
Petri Nets

True Concurrency

- Multiple transitions enabled
- Transitions fired one at a time



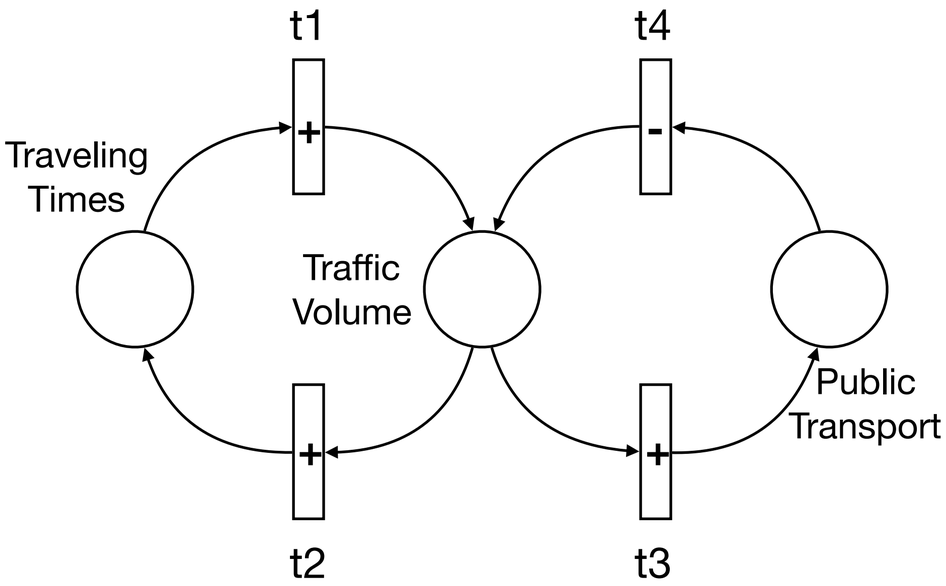
Causal Loop Nets



Direct correspondence

Variables as places

Links as transitions with polarity



Qualitative Abstractions

- **Qualitative values**

  Variables increase or decrease

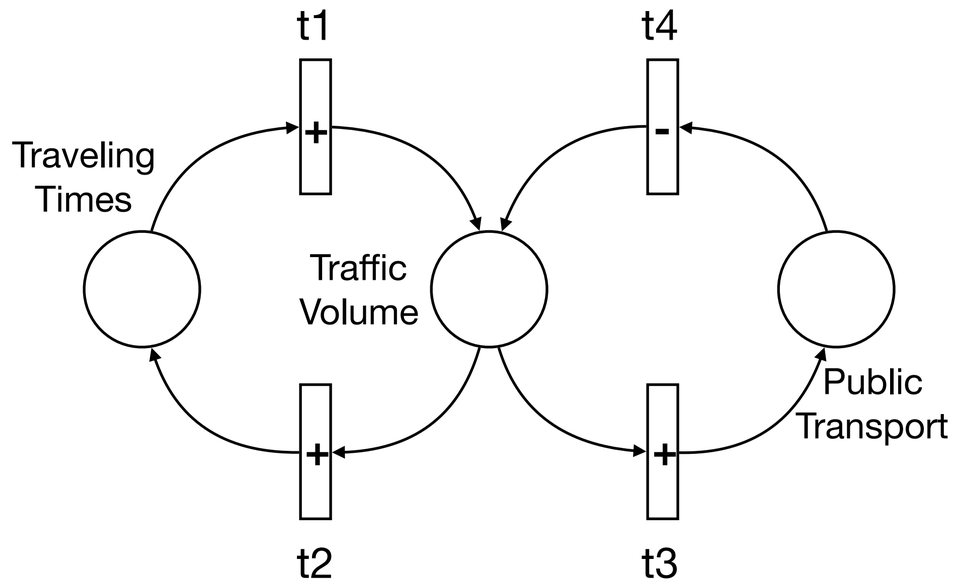
- **Delays**

  Delays are qualitative

  Tokens can be delayed

- **Concurrency**

  Tokens are not limited resources



Qualitative Abstractions

Qualitative values and non-deterministic delays

Variables increase or decrease

Tokens

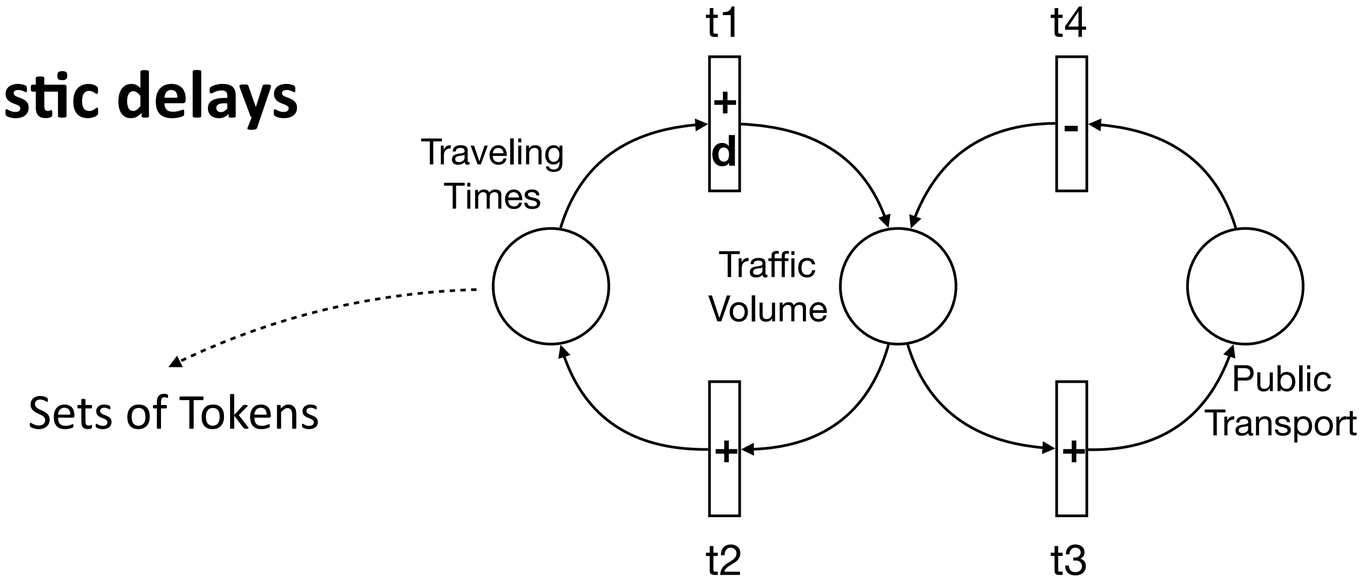
Delay $e \in \mathcal{N}_0$

$$Tokens = \{ \uparrow_e, \downarrow_e, \_ \}$$

No change or no information

Markings

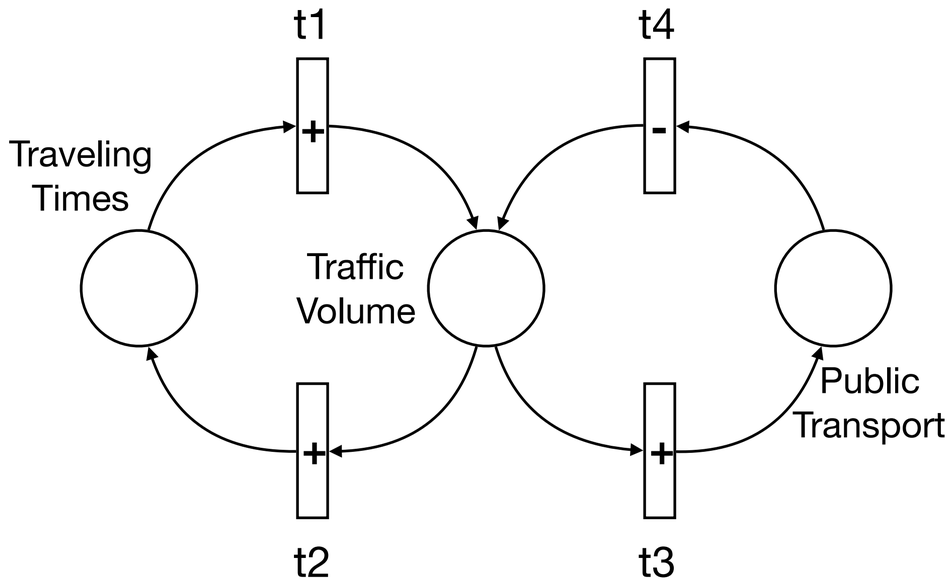
$$M : P \rightarrow 2^{Tokens}$$



Qualitative Abstractions

True concurrency and AMAN strategy

- All enabled transitions must fire
- As many tokens as needed

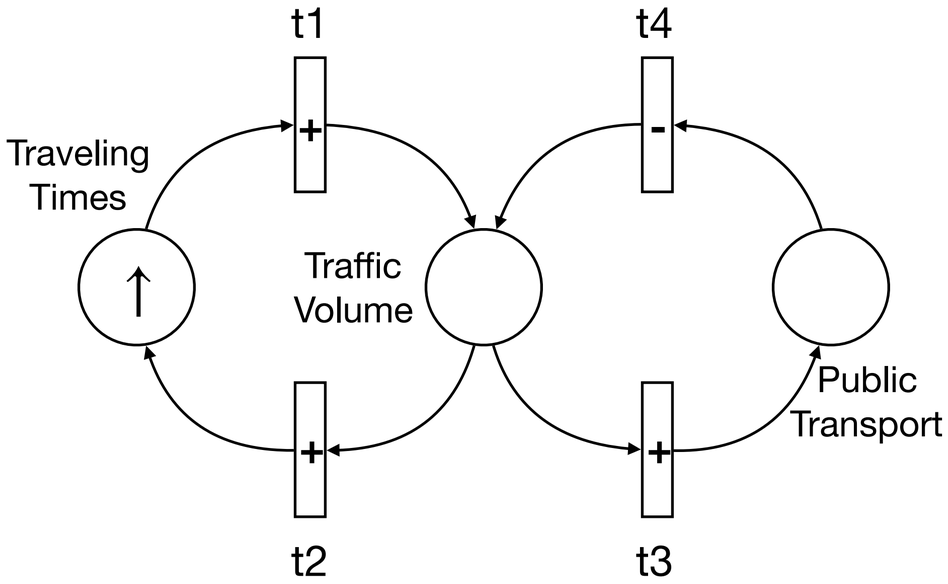


Enabled Transitions

All transitions with $\{ \uparrow_0 , \downarrow_0 \}$ in their incoming places

Causal Loop Nets

Concurrency



Marking Graph
(Semantics)

M_0 **(1,0,0)**

↓ t1

(0,1,0)

↓ t2|t3

(1,0,1)

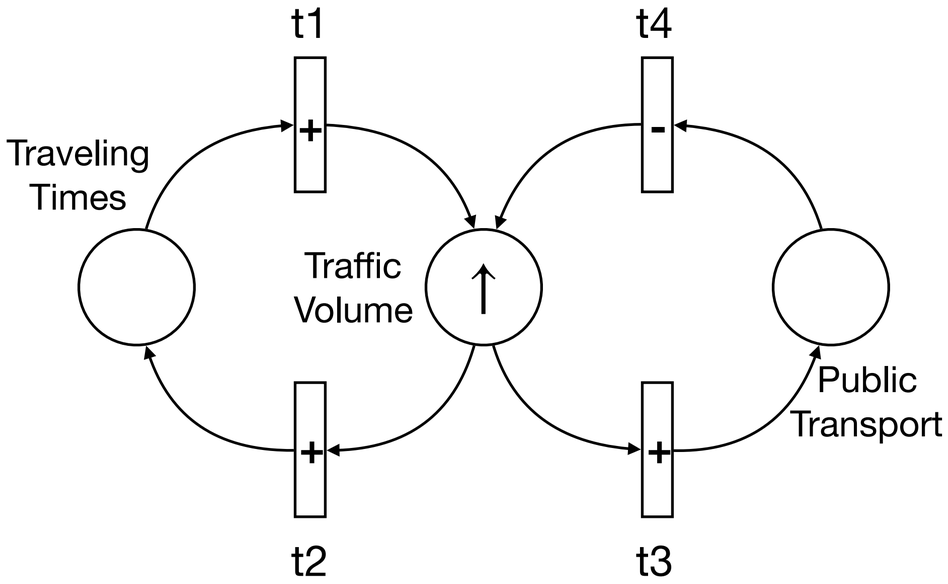
↓ t1|t4

(0,{1,-1},0)

↓ t2|t3

(?)

Concurrency



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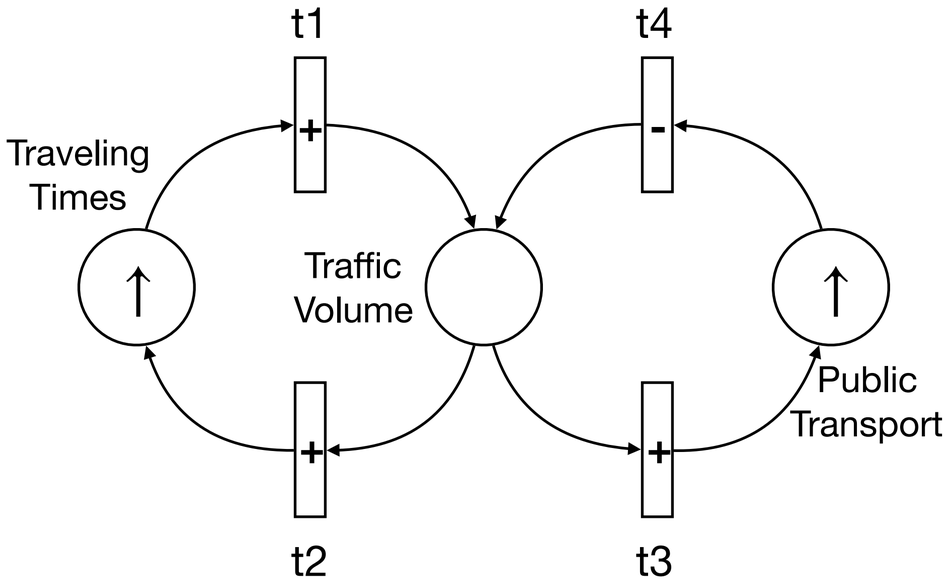

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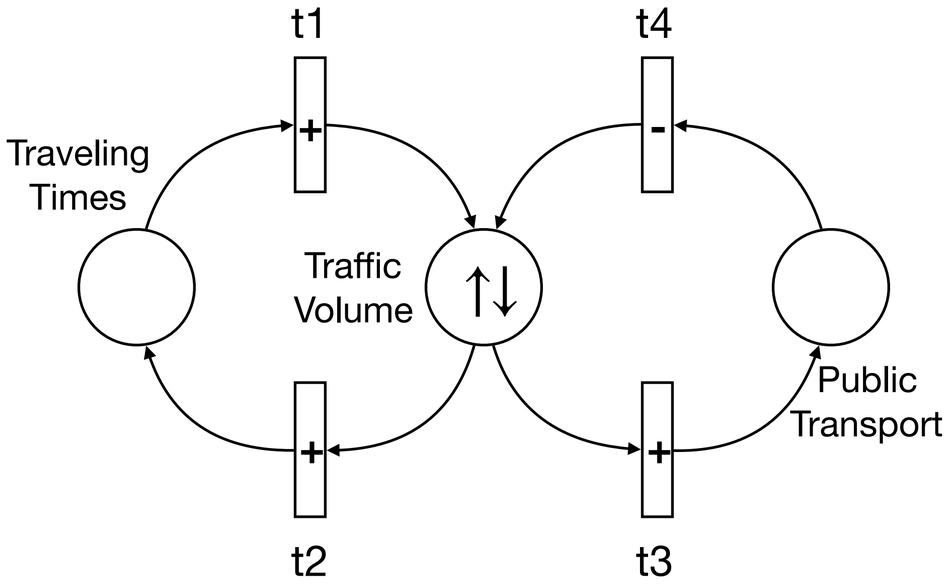

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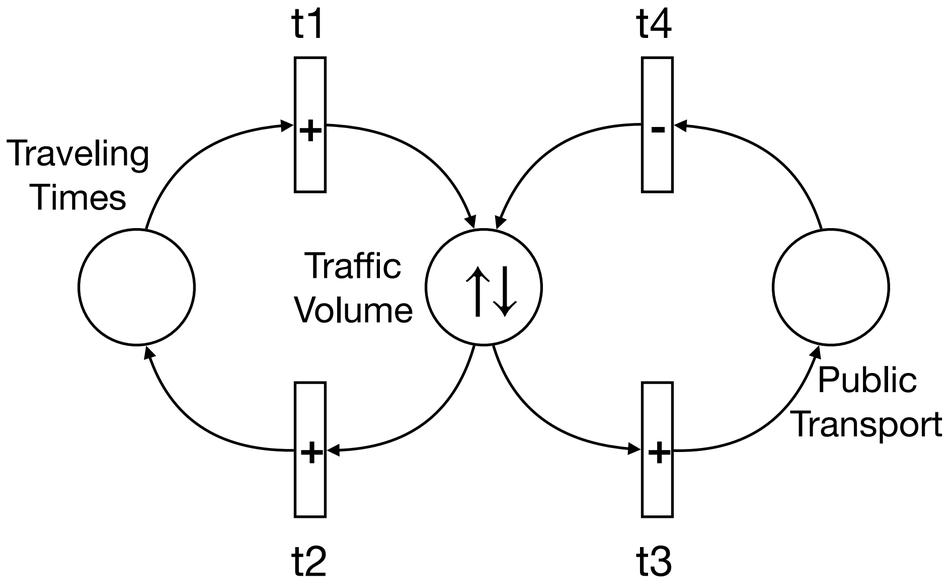

Concurrency

Normalize Marking Graph

Marking Graph
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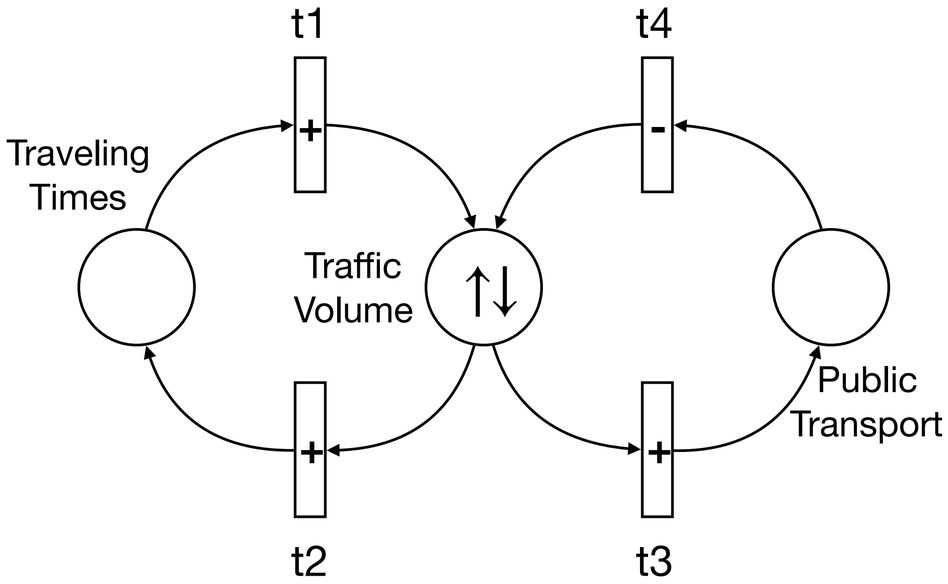

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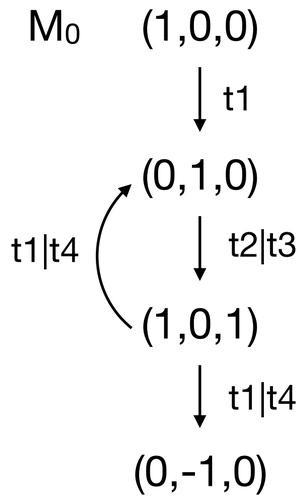

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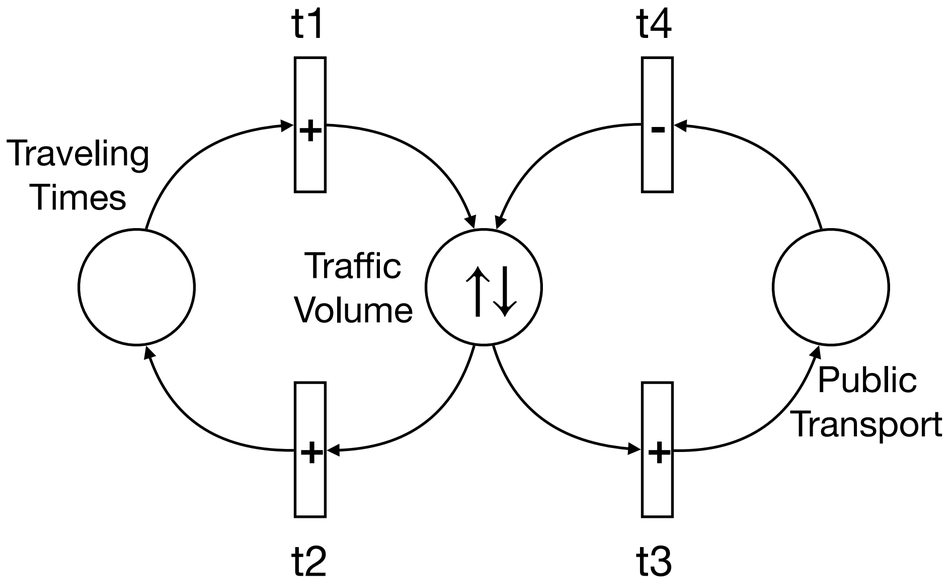

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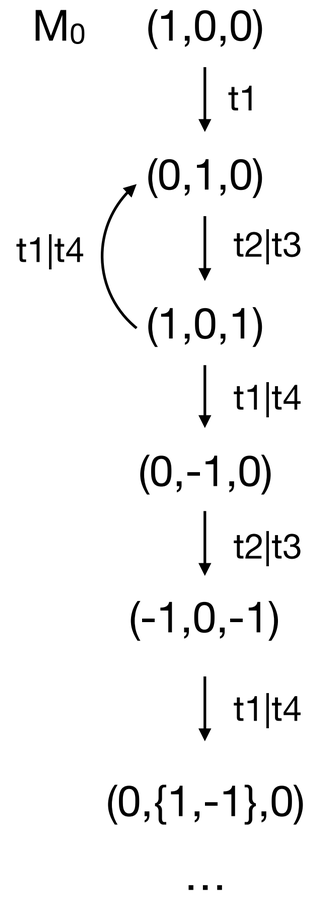

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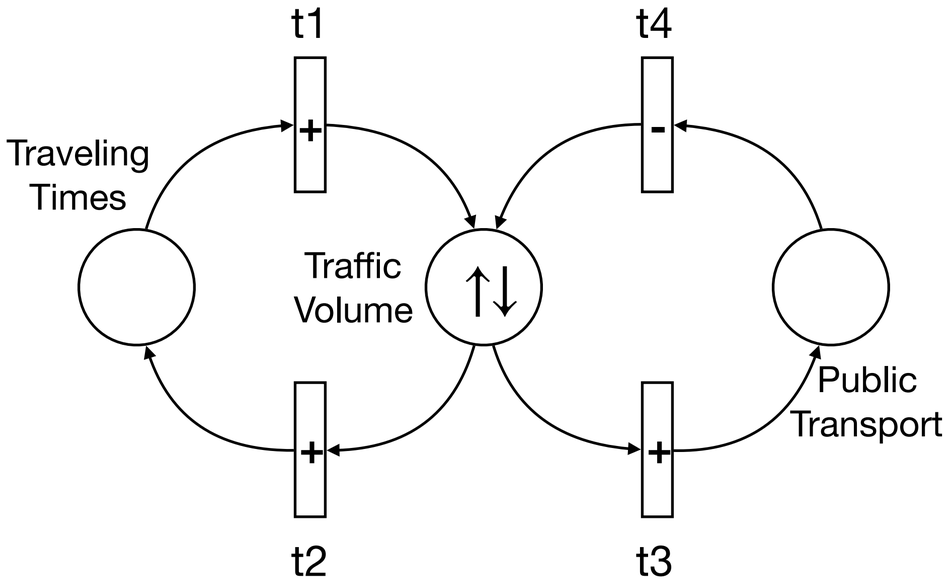

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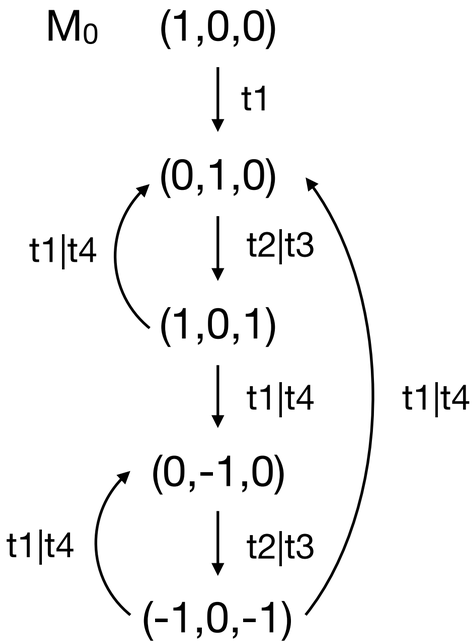

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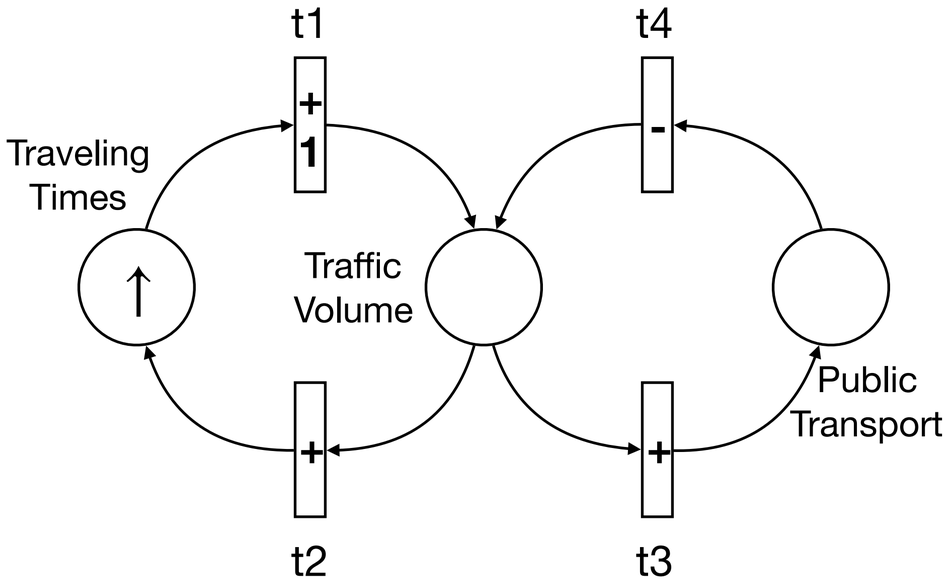

Delays

Each step is consider a **tick**

Delay tokens are decreased

Marking Graph
(Semantics)

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↓ t1

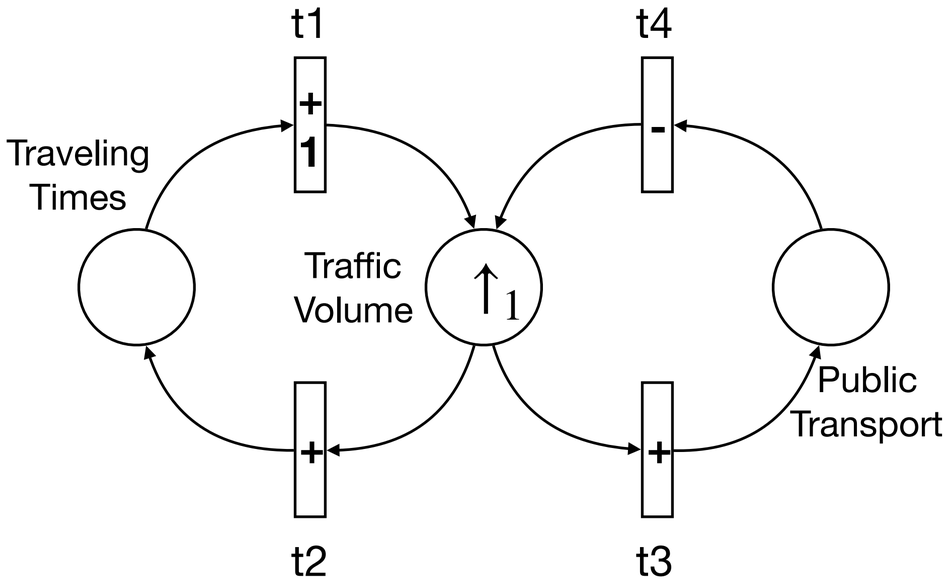
$(0,1_1,0)$

↓

(0,1,0)

Different Marking Graphs for different delays

Causal Loop Nets



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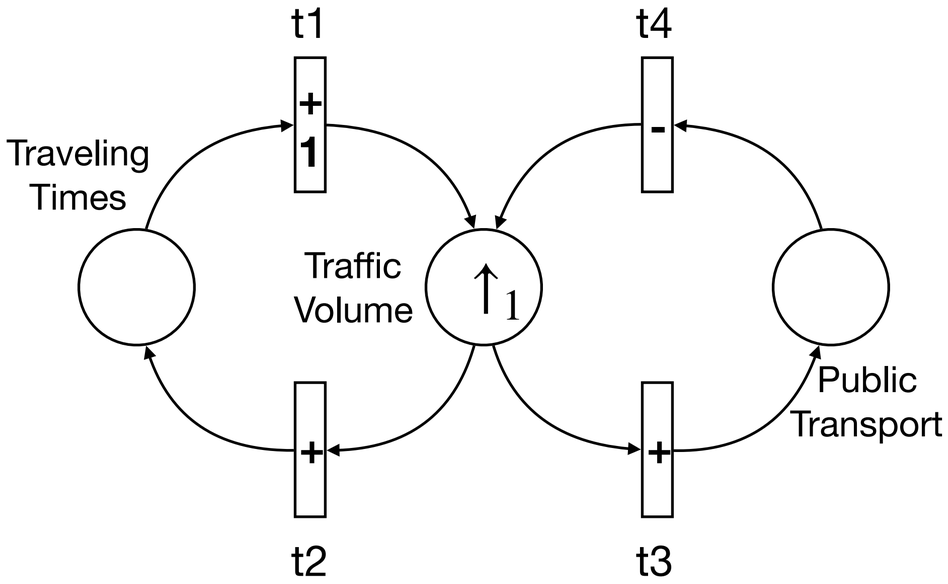

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↓ τ (silent transition)

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Different Marking Graphs for different delays

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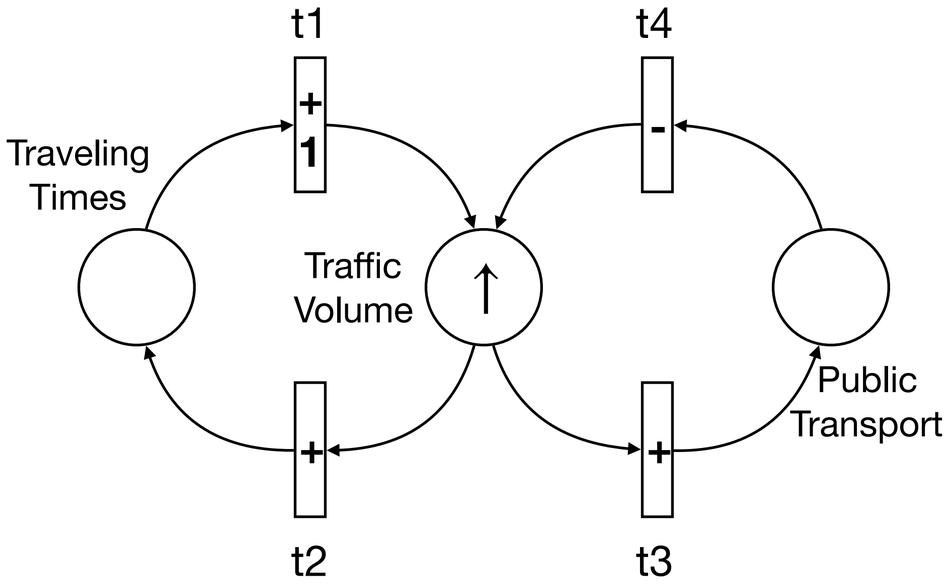

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$(0,1_1,0)$

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Different Marking Graphs for different delays

Analysis of Causal Loop Nets

Queries on traces (Sequence of Markings)

- Does a CLN exhibits a given behaviour? (Over a variable)
- How X behaves when Y satisfies some behaviour?

$$\varphi_0(i, r, v, f) = \bigwedge_{j=0}^{k-1} \exists s_{j+1} \,.\, (\sigma_i(s_j \,..\, s_{j+1}) \,.\, v \sim f(j))$$

$$\varphi_1(v, f) \;=\; \exists i, r : \;\; \varphi_0(i, r, v, f)$$

$$\varphi_3(\varphi_1(v^m, f), v^\ell, g^\ell)$$

Simulation relations

- One to one relation
- Abstract similar behaviour

$$\uparrow^n \sim \uparrow$$

$$\downarrow^n \sim \downarrow$$

$$\{ \uparrow, \downarrow \}\_ \sim \{ \uparrow, \downarrow \}$$

$$\uparrow \downarrow \nsim \uparrow \_ \downarrow$$

$$\uparrow \downarrow \sim \uparrow \_ \downarrow$$

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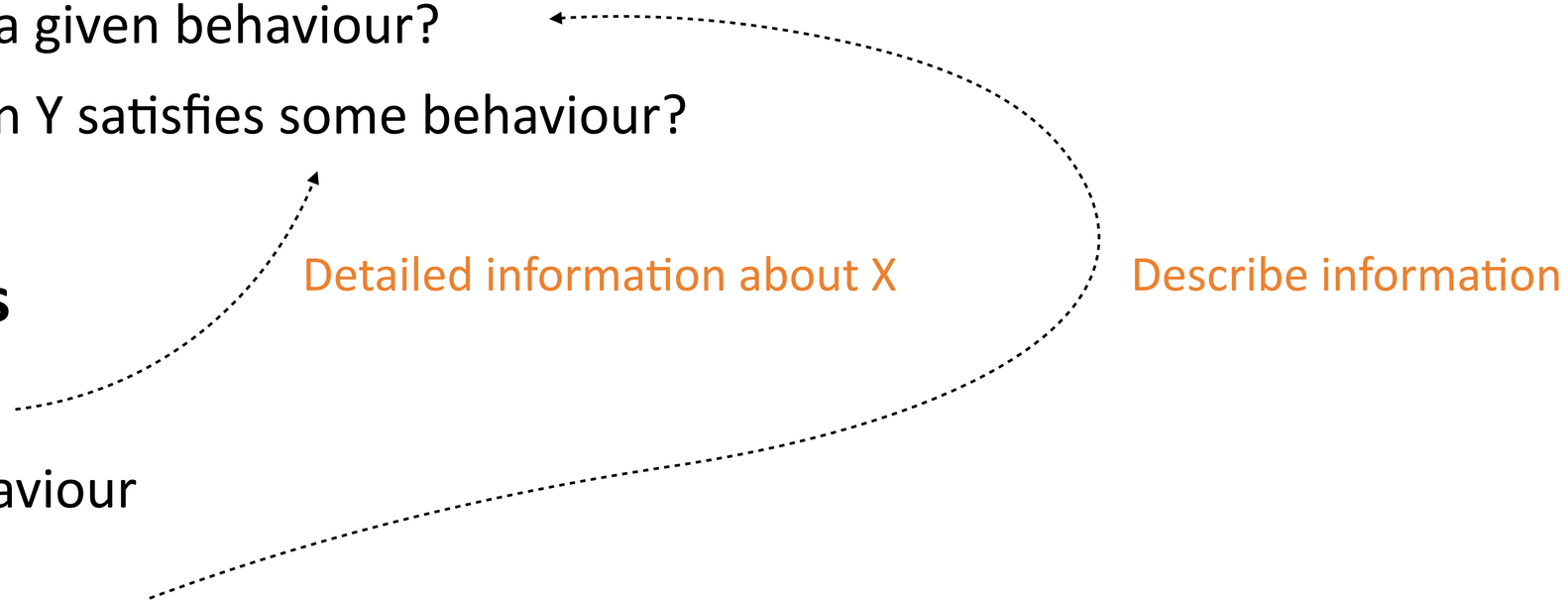
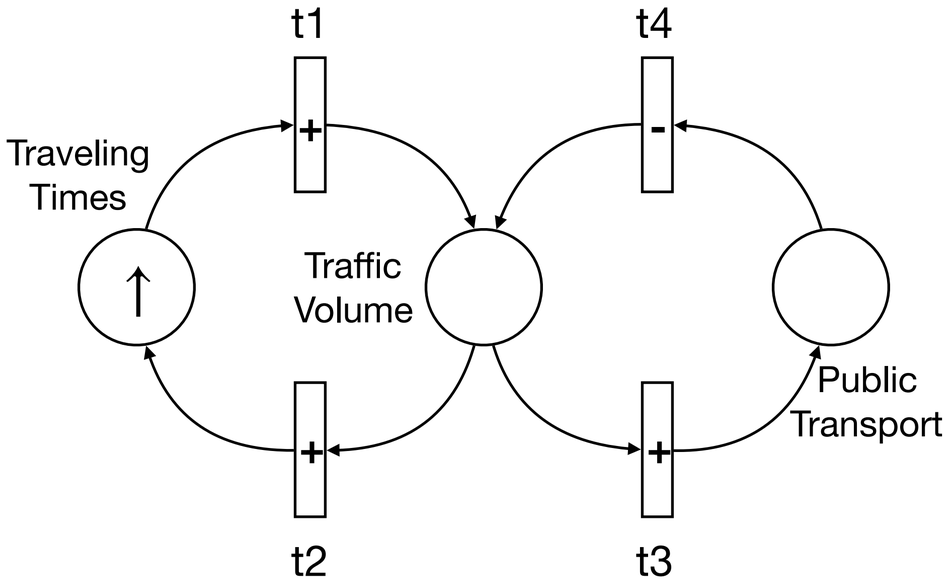


Analysis of Causal Loop Nets

Can Traveling Times eventually Increase and then Decrease?

$$\varphi_1(TravelingTimes, (\uparrow \downarrow))$$



true

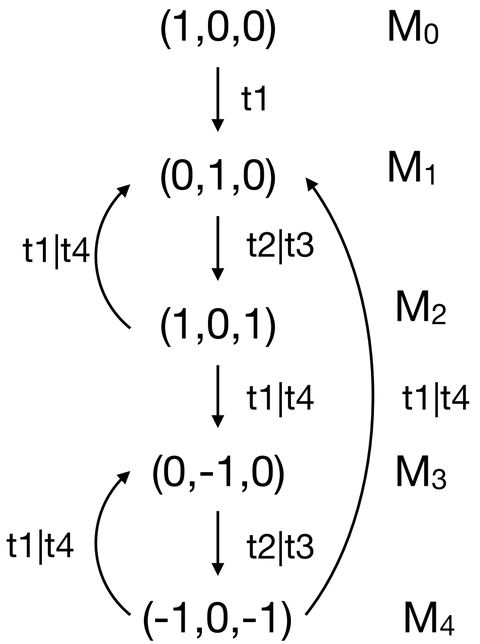
Tr1 = $M_0\, M_1\, M_2\, M_3\, M_4$

Tr2 = $M_0\, M_1\, M_2\, M_1\, M_2\, M_3\, M_4$

Tr3 = $M_0\, M_1\, M_2\, M_1\, M_2\, M_3\, M_4\, M_3\, M_4$

…

Marking Graph
(Semantics)



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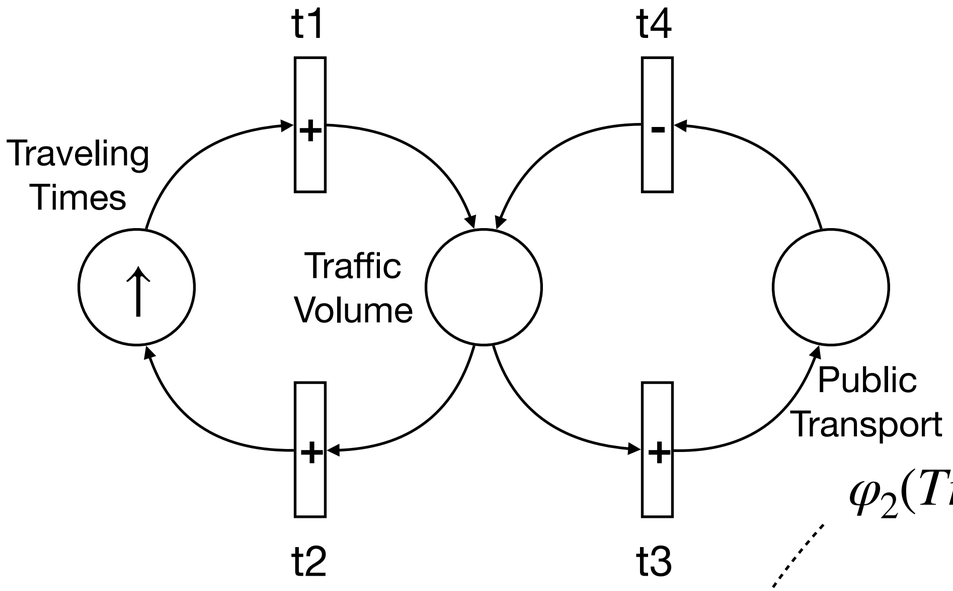
$$\varphi_1(TravelingTimes, (\uparrow \downarrow))$$



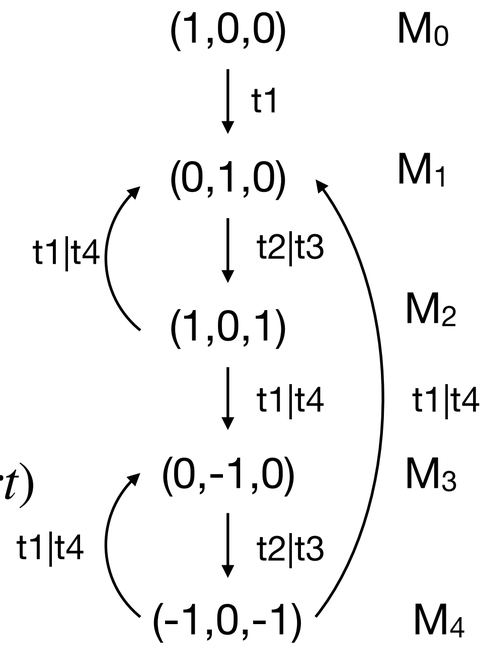
true Tr1 = $M_0\, M_1\, M_2\, M_3\, M_4$

Tr2 = $M_0\, M_1\, M_2\, M_1\, M_2\, M_3\, M_4$

Tr3 = $M_0\, M_1\, M_2\, M_1\, M_2\, M_3\, M_4\, M_3\, M_4$

…

Marking Graph
(Semantics)



$$\varphi_2(TravelingTimes, (\uparrow \downarrow), PublicTransport)$$

true Tr1 = _,1,0,-1,0

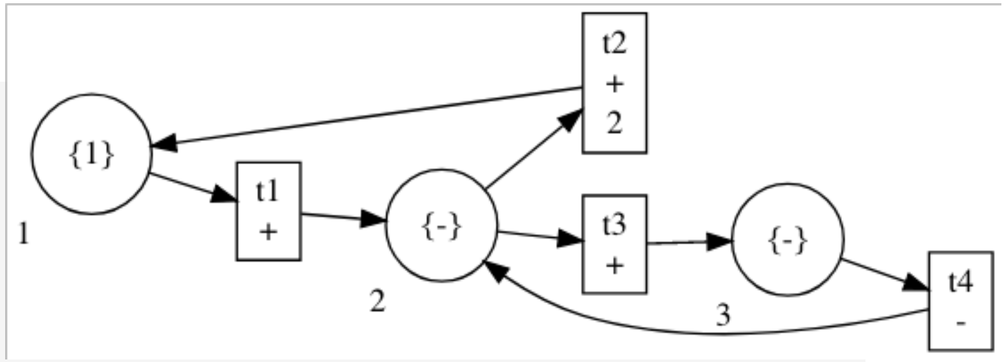
Tr2 = _,1,0,1,0,-1,0

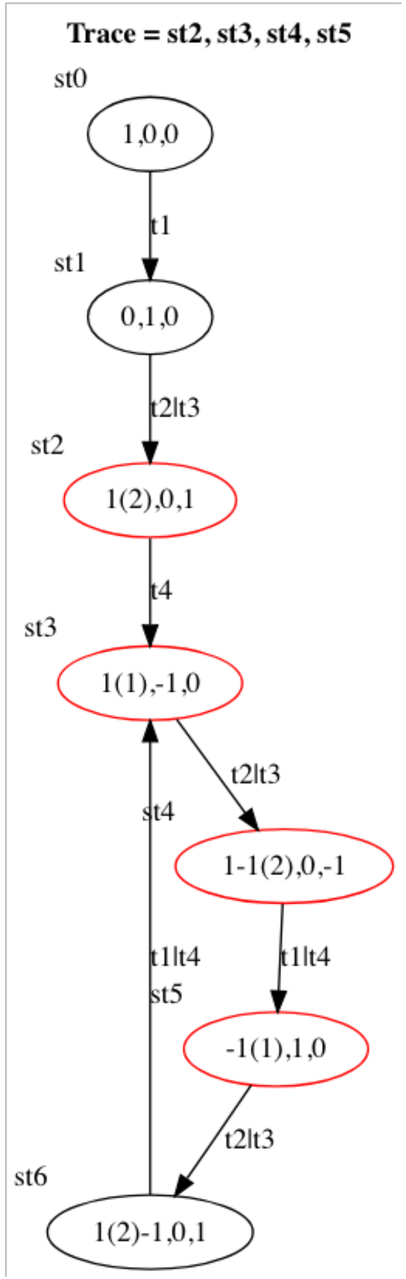
Tr3 = _,1,0,1,0,-1,0,-1,0

How Public Transport behaves when Traveling Times Increases and then Decreases?

Analysis of Causal Loop Nets

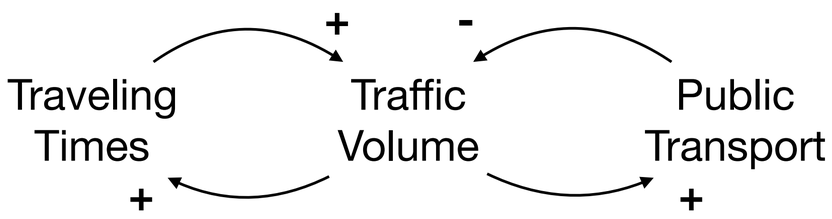
```
// Specify a CLN
  var traffic = newclpn ++ (
    1 -->+ 2 by "t1",
    2 -->+ 1 by "t2",
    2 -->+ 3 by "t3" in 2,
    3 -->- 2 by "t4"
  ) initMark((1, Set(1)))

// Graphical view of the Markin Graph
  reach2dot(traffic.behavior)

// Verification
  val trace = relaxedExists(traffic.behavior,1,List(Inc,Dec))

// Visualize Trace
  trace2dot(traffic.behavior,trace)
```





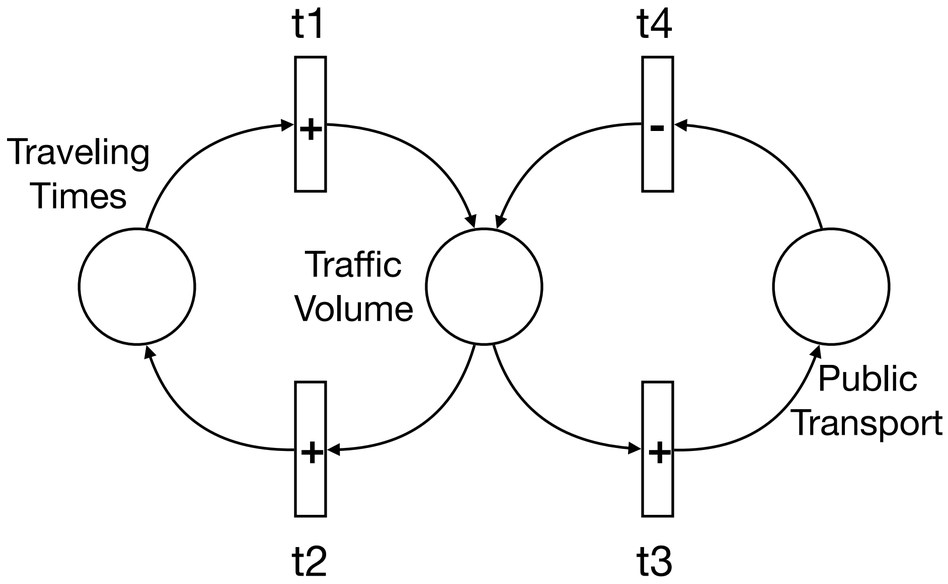
Wrapping up

CLD



- Informal semantics
- Difficult to analyse behaviour in complex systems
- Simulation is not exhaustive

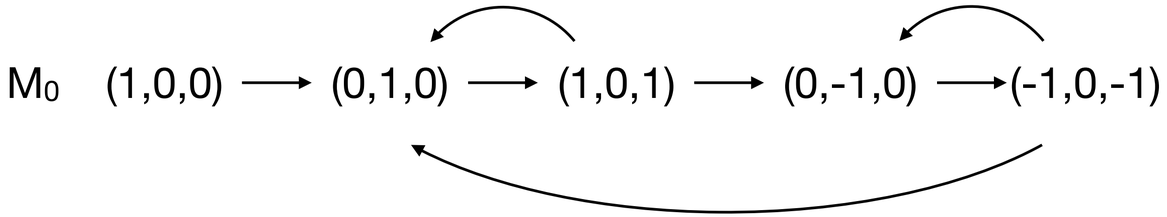
CLN



- Formal semantics
- Exhaustive analysis

Automatic generation

Marking Graph
(Semantics)



Thank you!